SECTION ONE: CALCULATOR-FREE 40 Marks

This section has **NINE (9)** questions Attempt **all** questions.

Question 1 [3 marks]

Simplify the following:

$$\frac{2a^3 - 7a^2 - 4a}{2a^2 - 5a - 3} \quad \times \quad \frac{6a-18}{16 - a^2}$$

Question 2 [1 + 1 + 2 = 4 marks]

Differentiate the following without simplifying:

(a) $y = \pi - x^3 + e^4$

(b) $y = e^{4x - 3x^2}$

(c) $y = \sqrt{4x^2 + 2x - 3}$

Question 3 [2 + 2 + 2 = 6 marks]

Given $f(x) = x^2 + 6$
$g(x) = \sqrt{x - 4}$
$h(x) = x^2 (x - 1)$

find:

(a) $f \circ g(x)$ expressing your answer in a simplified form

(b) the domain and range of $f \circ g(x)$

(c) the value(s) of x where $g \circ h(x)$ exists.

Question 4 [1 +1 + 2 = 4 marks]

Determine the following integrals:

(a) $\int \frac{2}{\sqrt{x}} - \sqrt[3]{x} \;\; dx$

(b) $\int_0^2 3(x + e^{3x}) \;\; dx$

(c) $\int \frac{x^3 - 1}{(x^4 - 4x)^3} \;\; dx$

Question 5 [5 marks]

A shopkeeper imports three varieties of fruit to sell in her shop The three varieties of fruit were apples, oranges and bananas The weight of apples was four kilograms less than eight times the weight of the oranges. The weight of apples was three times the total weight of the bananas and oranges

If the latest order of fruit was 80 kg, determine by setting up a system of equations how many kilograms of oranges were ordered

Question 6 [3 marks]

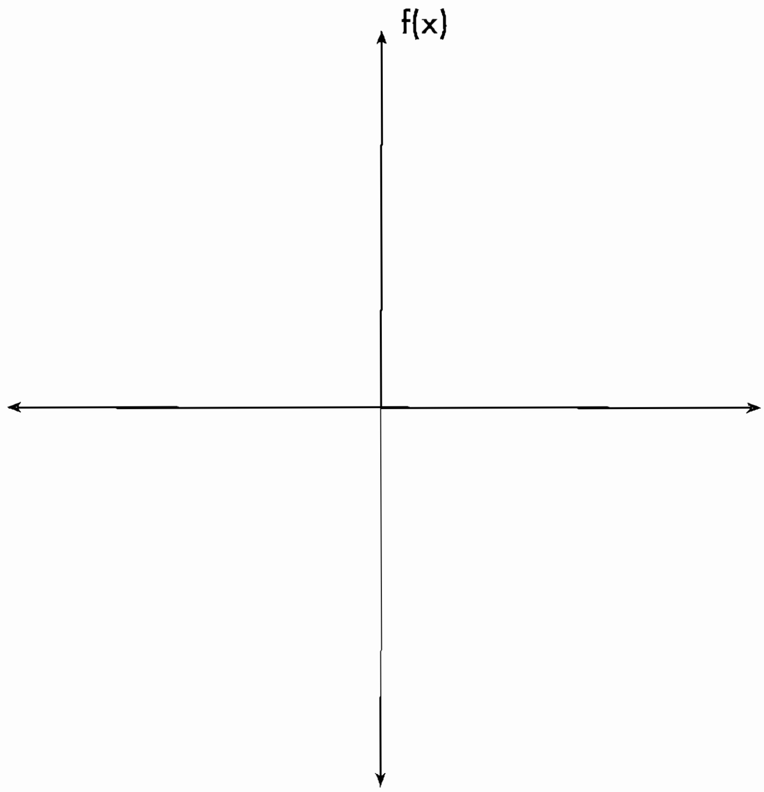
After investigating the addition of integers, Simon makes a conjecture that: 'The sum of two odd integers is even'

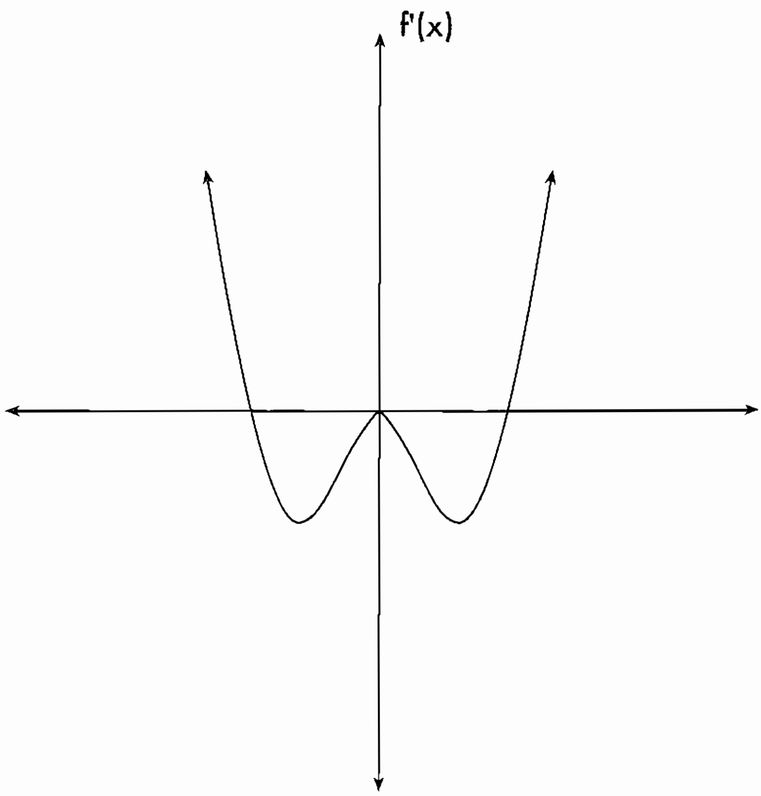
Is Simon correct? Prove using algebra

Question 7 [3 + 2 = 5 marks]

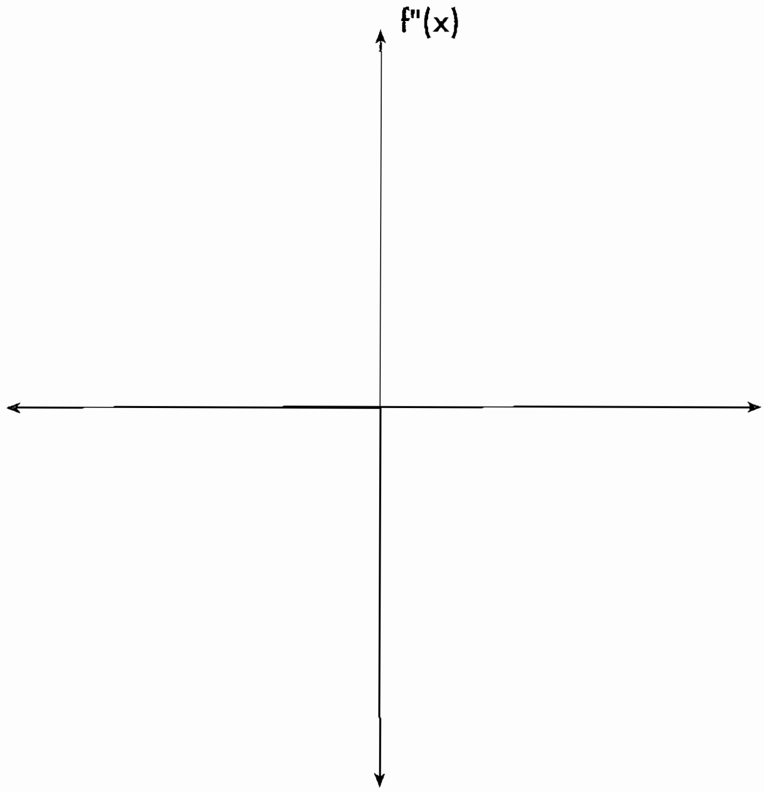
Sketch possible graphs of f(x) and f''(x) on the axes provided below given the graph of the derivative function f'(x)

(a)





(b)



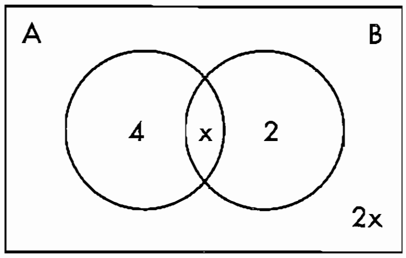
Question 8 [4 + 2 = 6 marks]

(a) Determine all turning points, their nature and points of inflection for the function $f(x) = x^3 - 3x^2$

(b) Find the maximum and minimum values of the function $f(x) = x^3 - 3x^2$ over the interval $-2 \leq x \leq 1$

Question 9 [1 + 3 = 4 marks]

Given the following Venn diagram showing events A and B



Determine x if:

(a) A and B are mutually exclusive

(b) A and B are independent

SOLUTIONS TO TRIAL PAPER

Section One: Calculator Free

1 $\dfrac{(2a^3 - 7a^2 - 4a)}{(2a^2 - 5a - 3)} \times \dfrac{(6a - 18)}{(16 - a^2)}$

$= \dfrac{a\cancel{(2a+1)}(a-4)}{\cancel{(2a+1)}\cancel{(a-3)}} \times \dfrac{6\cancel{(a-3)}}{(4-a)(4+a)}$ ✓

$= \dfrac{6a(-a+4)}{(4-a)(4+a)}$

$= \dfrac{6a\cancel{(-a-4)}}{\cancel{(4-a)}(4+a)}$ ✓

$= \dfrac{-6a}{(4+a)}$ ✓

2

(a) $y = \pi - x^3 + e^4$

$\dfrac{dy}{dx} = -3x^2$ ✓

(b) $y = e^{4x - 3x^2}$

$\dfrac{dy}{dx} = (4 - 6x)e^{4x - 3x^2}$ ✓

(c) $y = (4x^2 + 2x - 3)^{½}$

$\dfrac{dy}{dx} = \dfrac{1}{2}(4x^2 + 2x - 3)^{-½}(8x + 2)$ ✓✓

3.

(a) $f(\sqrt{x-4})$

$= (\sqrt{x-4})^2 + 6$ ✓

$= x + 2$ ✓

(b) $D_{fog} = \{x : x \geq 4\}$ ✓

$R_{fog} = \{y : y \geq 6\}$ ✓

(c) Value(s) of x where goh(x) exist is $x \geq 2$ ✓✓

4

(a) $\displaystyle\int \frac{2}{\sqrt{x}} - \sqrt[3]{x} \quad dx$

$= \displaystyle\int 2x^{-\frac{1}{2}} - x^{\frac{1}{3}} \quad dx$

$= 4x^{\frac{1}{2}} - \dfrac{3x^{\frac{4}{3}}}{4} + c$ ✓

(b) $\displaystyle\int_0^2 3(x + e^{3x})\,dx$

$= \displaystyle\int_0^2 (3x + 3e^{3x})\,dx$

$= \left[\dfrac{3x^2}{2} + e^{3x}\right]_0^2$

$= (6 + e^6) - (0 + 1)$

$= e^6 + 5$ ✓

(c) $\displaystyle\int \frac{x^3 - 1}{(x^4 - 4x)^3}\,dx$

$= \dfrac{1}{4}\displaystyle\int \frac{(4x^3 - 4)}{(x^4 - 4x)^3}\,dx$ ✓

$= \dfrac{1}{4}\displaystyle\int (x^4 - 4x)^{-3}(4x^3 - 4)\,dx$

$= \dfrac{1}{4}\;\dfrac{(x^4 - 4x)^{-2}}{(-2)} + c$

$= -\dfrac{1}{8(x^4 - 4x)^2} + c$ ✓

5.

Let
a = apples
b = bananas
c = oranges

$a + b + c = 80$ ✓
$a = 8c - 4$ ✓
$a = 3(b + c)$ ✓

$\therefore$ $8c - 4 + b + c = 80$
$8c - 4 - 3b - 3c = 0$

$\therefore$ $9c + b = 84$ ①
$5c - 3b = 4$ ② ✓

$27c + 3b$	$= 252$	① x 3 = ③
$5c - 3b$	$= 4$	②
$32c$	$= 256$	③+②
c	$= 8$ ✓	

$\therefore$ There were 8 kg of oranges

6.

Let m and n be the odd integers

$m = 2x + 1$

$n = 2y + 1$ ✓ for some integers x and y

$\therefore m + n = (2x + 1) + (2y + 1)$

$= 2x + 2y + 2$

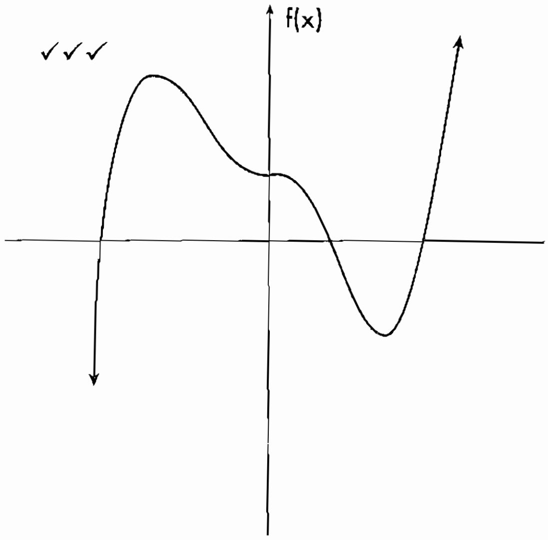
$= 2(x + y + 1)$ ✓

Since $x + y + 1$ is an integer and $2(x + y + 1)$ is divisible by 2, $m + n$ is divisible by 2 ✓

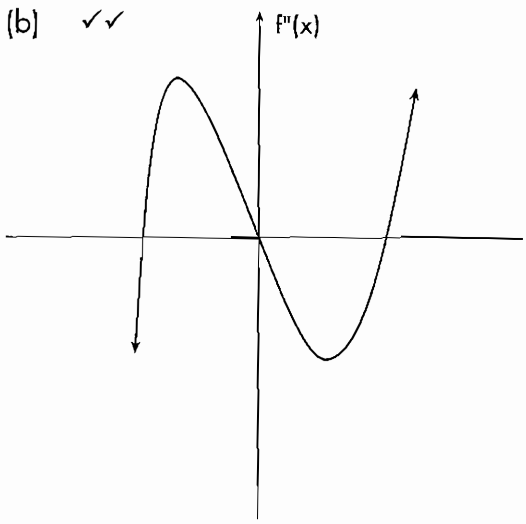
Hence the sum of two odd integers is even

7

(a) ✓✓✓



(b) ✓✓



8 (a) $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

Turning points when $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x - 2) = 0$

$\therefore x = 0,\ x = 2$

(0,0) (2,–4)
✓ ✓

Nature:

when $x = 0$ $f''(0) = -6$ $\therefore$ maximum

when $x = 2$ $f''(2) = 6$ $\therefore$ minimum ✓

Points of inflection when $f''(x) = 0$

$\therefore 6x - 6 = 0$

$x = 1$

(1,–2) ✓

(b) Values will occur at end points or critical points.

Max : (0,0)

Min : (2, –4)

$f(-2) = -20$

$f(1) = -2$

Maximum value : $f(0) = 0$ ✓

Minimum value : $f(-2) = -20$ ✓

9.

(a) $P(A \cap B) = 0$ for mutually exclusive events

$\therefore x = 0$ ✓

(b) $P(A \cap B) = P(A)\,P(B)$ for independent events

$$\frac{(x)}{(6 + 3x)} = \frac{(4 + x)}{(6 + 3x)} \bullet \frac{(2 + x)}{(6 + 3x)} \checkmark$$

$$\frac{(6 + 3x)^{2}(x)}{(6 + 3x)} = (4 + x)(2 + x)$$

$6x + 3x^2 = 8 + 6x + x^2$ ✓

$\therefore 2x^2 = 8$